What we have learned from direct CP violation studies in kaon decays

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A self-consistent analysis of $K \to 2\pi$ and $K \to 3\pi$ decays within a unique framework of chiral dynamics applied to the QCD-corrected weak nonleptonic quark lagrangian has been performed. The results on $K \to 2\pi$ amplitudes at $O(p^6)$, including the value for $\varepsilon^{'}/\varepsilon$, are compared with experiment to fix phenomenological B-factors for mesonic matrix elements of nonpenguin and penguin four-quark operators. The dependence of these B-factors on different theoretical uncertainties and experimental errors of various input parameters is investigated. Finally, we present our estimates at $O(p^6)$ for the CP asymmetry of linear slope parameters in the $K^{\pm} \to 3\pi$ Dalitz plot.

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1 Introduction

The starting point for most calculations of nonleptonic kaon decays is an effective weak lagrangian of the form [1, 2]

$$\mathcal{L}_w^q\big(|\Delta S| = 1\big) = \sqrt{2}\, G_F\, V_{ud} V_{us}^* \sum_i \widetilde{C}_i\, \mathcal{O}_i\,, \tag{1}$$

which can be derived with the help of the Wilson operator product expansion (OPE) from elementary quark processes, with additional hard gluon exchanges. In the framework of perturbative QCD the coefficients $\widetilde{C}_i$ are to be understood as scale and renormalization scheme dependent functions. There exist extensive next-to-leading order (NLO) calculations [3, 4] in the context of kaon decays, among others. These calculations are based on the possibility of factorization of short- and long-distance contributions into Wilson coefficient functions $C_i^{QCD}(\mu)$ and mesonic matrix elements of four-quark operators $\mathcal{O}_i$, respectively. The latter, however, can presently be obtained only by using nonperturbative, i.e. model-dependent, methods, because not only perturbative QCD breaks down at scales $\mu \leq 1$GeV, but also the QCD degrees of freedom (quarks and gluons) have to be replaced by the mesonic ones.

Usually, the results of calculations are displayed with the help of B-factors (bag

parameters) in the form

$$T_{K\to 2\pi} = \sqrt{2}\, G_F\, V_{ud}V_{us}^{*} \sum_i \Big[C_i(\mu)B_i(\mu) \Big] < \pi\pi|\mathcal{O}_i|K >_{vac.sat.}\,, \qquad (2)$$

where the mesonic matrix elements of four-quark operators are approximated by their vacuum saturation values, which are real and μ-independent. In principle, factors $B_i(\mu)$ should be estimated by some higher-order calculations in the long-distance regime, for instance, in $1/N_c$-expansion [5] in the form $1 + O(1/N_c)$, or from the lattice approach. The preliminary stage of these calculations is best characterized by the long standing difficulties to explain quantitatively the well-known $\Delta I = 1/2$ rule. Recent lattice calculations [6] seem to succeed in this respect, but are at the same time at variance (even in sign) with experimental values of $\mathrm{Re}(\varepsilon^{'}/\varepsilon)$. Of course, the severe difficulties of all calculations of long-distance effects from "first principles" restricts the predictive power of (1), leaving only the possibility of some semi-phenomenological treatment [3, 7, 8], which cannot be used to test the Standard Model (SM) in the kaon sector in a fundamental way.

The main aim of the present paper is a further semi-phenomenological analysis of the long-distance (nonperturbative) aspects of the above lagrangian, especially in view of the actuality of the task to analyze the implications of the measured parameter of direct CP violation, $\varepsilon^{'}/\varepsilon$, on an alternative manifestation of direct CP violation by asymmetries in charged $K^{\pm} \to 3\pi$ decays. As the main result, we present numerical estimates of bag parameters and resulting asymmetries, in the form of probability densities, in order to demonstrate the experimental and theoretical uncertainties.

2 General scheme of calculations

In (1), $\mathcal{O}_i$ are the four-quark operators, defined either by combinations of products of quark currents ($i = 1, 2, 3, 4$, non-penguin diagrams) or, in case of gluonic ($i = 5, 6$) and electro-weak ($i = 7, 8$) penguin operators, by products of quark densities. The operators $\mathcal{O}_i$ with $i = 1, 2, 3, 5, 6$ describe weak transitions with isospin change $\Delta I = 1/2$ while the operator $\mathcal{O}_4$ corresponds to a $\Delta I = 3/2$ transition and operators $\mathcal{O}_{7,8}$ – to mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes.

The operators $\mathcal{O}_i$ used here and in earlier work [9, 10, 11] differ from those used in [3, 5, 7, 8] and other papers [4, 12, 13, 14] (usually called Q_i). Both sets are related by linear relations, which are given for easy reference below:

$$\begin{aligned}
&Q_1 = 2\mathcal{O}_1 + \frac{2}{5}\mathcal{O}_2 + \frac{4}{15}\mathcal{O}_3 + \frac{4}{3}\mathcal{O}_4\,, \\
&Q_2 = -2\mathcal{O}_1 + \frac{2}{5}\mathcal{O}_2 + \frac{4}{15}\mathcal{O}_3 + \frac{4}{3}\mathcal{O}_4\,, \\
&Q_3 = 2\mathcal{O}_1 + 2\mathcal{O}_2\,, \\
&Q_4 = -2\mathcal{O}_1 + 2\mathcal{O}_2\,, \\
&Q_5 = 4\mathcal{O}_6\,, \qquad Q_6 = 2\mathcal{O}_5 + \frac{4}{3}\mathcal{O}_6\,,
\end{aligned}$$

$$Q_7 = \mathcal{O}_7 \,, \qquad Q_8 = \frac{1}{2}\mathcal{O}_8 + \frac{1}{3}\mathcal{O}_7 \,.$$

The general scheme of the meson matrix element calculation by using the weak lagrangian (1) is based on the quark bosonization approach [10]. The bosonization procedure establishes a correspondence between quark and meson currents (densities) and products of currents (densities). Finally, it leads to an effective lagrangian for nonleptonic kaon decays in terms of bosonized (meson) currents and densities:

$$\bar{q}\gamma_\mu \frac{1}{4}(1 \mp \gamma^5)\lambda^a q \;\Rightarrow\; J^{a\,(mes)}_{L/R\,\mu} \,, \quad \bar{q}\frac{1}{4}(1 \mp \gamma^5)\lambda^a q \;\Rightarrow\; J^{a\,(mes)}_{L/R} \,.$$

The meson currents/densities $J^a_{L/R\mu}$ and $J^a_{L/R}$ are obtained from the quark determinant by variation over additional external sources associated with the corresponding quark currents and densities [10]. From the momentum expansion of the quark determinant to $O(p^{2n})$ one can derive the strong lagrangian for mesons $\mathcal{L}_{eff}$ of the same order and the corresponding currents and densities $J^a_{L/R\mu}$ and $J^a_{L/R}$ to the order $O(p^{2n-1})$ and $O(p^{2n-2})$, respectively. Thus, the bosonization approach gives us the correspondence between power counting for the momentum expansion of the effective chiral lagrangian of strong meson interactions,

$$\mathcal{L}^{(mes)}_s = \mathcal{L}^{(p^2)}_s + \mathcal{L}^{(p^4)}_s + \mathcal{L}^{(p^6)}_s + \ldots ,$$

and power counting for the meson currents and densities:

$$\mathcal{L}^{(p^n)}_s \;\Rightarrow\; J^{(p^{n-1})}_\mu \text{ (currents)} \,; \quad \mathcal{L}^{(p^n)}_s \;\Rightarrow\; J^{(p^{n-2})} \text{ (densities)} \,.$$

Some interesting observations on the difference of the momentum behavior of penguin and non-penguin operators can be drawn from power-counting arguments. The leading contributions to the vector currents and scalar densities are of $O(p^1)$ and $O(p^0)$, respectively. Since in our approach the non-penguin operators are constructed out of the products of currents $J^a_{L\mu}$, while the penguin operators are products of densities J^a_L, the lowest-order contributions of non-penguin and penguin operators are of $O(p^2)$ and $O(p^0)$, respectively. However, due to the well-known cancellation of the contribution of the gluonic penguin operator $\mathcal{O}_5$ at the lowest order [15], the leading gluonic penguin as well as non-penguin contributions start from $O(p^2)$ [1]). Consequently, in order to derive the currents which contribute to the non-penguin transition operators at the leading order, it is sufficient to use the terms of the quark determinant to $O(p^2)$ only. At the same time the terms of the quark determinant to $O(p^4)$ have to be kept for calculating the penguin contribution at $O(p^2)$, since it arises from the combination of densities, which are of $O(p^0)$ and $O(p^2)$, respectively. In this subtle way the difference in momentum behavior is revealed between matrix elements for these two types of weak transition operators; it manifests itself more drastically in higher-order lagrangians and currents.

[1]) There is no cancellation of the contribution of the electromagnetic penguin operator $\mathcal{O}_8$ at the lowest order and the leading contributions start in this case from $O(p^0)$

This fact makes penguins especially sensitive to higher order effects. In particular, the difference in the momentum power counting behavior between penguin and non-penguin contributions to the isotopic amplitudes of $K \to 3\pi$ decays, which appears in higher orders of chiral theory, leads to a dynamical enhancement of the charge asymmetry of the Dalitz-plot linear slope parameter [9, 10].

In our approach the Wilson coefficients $\widetilde{C}_i$ in the effective weak lagrangian (1) are treated as phenomenological parameters which should be fixed from experiment. They are related with the Wilson coefficients $C_i^{QCD}(\mu)$, calculated in perturbative QCD [7], via the $\widetilde{B}_i$-factors at the fixed renormalization scale $\mu = 1$ GeV:

$$\widetilde{C}_i = C_i^{QCD}(\mu)\widetilde{B}_i(\mu)|_{\mu=1GeV} .$$

The coefficients $C_i^{QCD}(\mu)$ can be written in the form of a sum of z and y components,

$$\widetilde{C}_i^{QCD}(\mu) = \widetilde{C}_i^{(z)}(\mu) + \tau\, \widetilde{C}_i^{(y)}(\mu)\,, \quad \tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\,, \tag{3}$$

where only the y component contains the combination of CKM-matrix elements τ, which, according to the SM, is responsible for CP violation – it has a non-vanishing imaginary part. For the amplitudes of nonleptonic kaon decays follows the same decomposition into z- and y- components,

$$\mathcal{A}_I = \mathcal{A}_I^{(z)} + \tau\mathcal{A}_I^{(y)}\,, \tag{4}$$

where $\mathcal{A}_I$ are isotopic amplitudes of $K \to 2\pi$ decay corresponding to the $\pi\pi$ final states with the isospin $I = 0, 2$.

With the dominating contributions $\mathcal{A}_I^{(i)}$ from the four-quark operators $\mathcal{O}_i$, the $K \to (2\pi)_I$ amplitudes may be written as

$$\begin{aligned}\mathcal{A}_I^{(z,y)} \quad = \quad & \Big[- C_1^{(z,y)}(\mu) + C_2^{(z,y)}(\mu) + C_3^{(z,y)}(\mu)\Big] \widetilde{B}_1(\mu)\,\mathcal{A}_I^{(1)} \\ & + C_4^{(z,y)}(\mu)\widetilde{B}_4(\mu)\,\mathcal{A}_I^{(4)} + C_5^{(z,y)}(\mu)\widetilde{B}_5(\mu)\,\mathcal{A}_I^{(5)} + C_8^{(z,y)}(\mu)\widetilde{B}_8(\mu)\,\mathcal{A}_I^{(8)}\,.\end{aligned}$$

In the calculation of the $\mathcal{A}_I^{(i)}$, sizable corrections connected to isospin symmetry breaking have been taken into account (see below).

The observable effects of direct CP violation in the nonleptonic kaon decays are caused by the y-components of their amplitudes (4). In particular, the ratio ε'/ε can be expressed as

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Im}\,\lambda_t\,\big(P_0 - P_2\big), \quad P_I = \frac{\omega}{\sqrt{2}|\varepsilon||V_{ud}||V_{us}|}\,\frac{\mathcal{A}_I^{(y)}}{\mathcal{A}_I^{(z)}}\,,$$

where $\mathrm{Im}\,\lambda_t = \mathrm{Im}\,V_{ts}^* V_{td} = |V_{ub}||V_{cb}|\sin\delta$; $\omega = \mathcal{A}_2^{(z)}/\mathcal{A}_0^{(z)}$.

3 Theoretical and phenomenological uncertainties

There are the following theoretical and phenomenological uncertainties which appear both from short distance (Wilson coefficients) and long distance (effective chiral lagrangians and $\widetilde{B}_i$-factors) contributions to the kaon decay amplitude:

– Dependence on the parameter $\mathrm{Im}\,\tau \sim \mathrm{Im}\,\lambda_t$ arising from the imaginary part of Wilson coefficient (3).

– Regularization scheme dependence which arises when calculating Wilson coefficients beyond the leading order (LO) of QCD in the next-to-leading orders: naive dimensional regularization (NDR), t'Hooft-Veltman regularization (HV).

– Dependence of Wilson coefficients on choice of the renormalization point μ, taken below as 1 GeV, and QCD scale $\Lambda^{(4)}_{\overline{MS}}$, contained in the interval (325 ± 110) MeV.

– Factors $\widetilde{B}_i$ $(i = 1, 4, 5, 8)$ for dominating contributions of four-quark operators $\mathcal{O}_i$ to the meson matrix element (5).

– Dependence of meson matrix elements on the structure constants L_2, L_3, L_4, L_5, L_8 of the general form of the effective chiral lagrangian introduced at $O(p^4)$ by Gasser and Leutwyler [16].

– Dependence of meson matrix elements on the structure constants of the effective chiral lagrangian at $O(p^6)$ [17].

– Dependence on choice of the regularization scheme to fix the UV divergences resulting from meson loops.

In the present paper we have combined a new systematic calculation of mesonic matrix elements for nonleptonic kaon decays from the effective chiral lagrangian approach with Wilson coefficients $C_i^{QCD}(\mu)$, derived by the Munich group [7] for $\mu = 1$ GeV and $m_t = 170$ GeV. For the parameter $\mathrm{Im}\,\lambda_t$ we have used the result obtained in [18]: $\mathrm{Im}\,\lambda_t = (1.33 \pm 0.14) \cdot 10^{-4}$. We performed a complete calculation of $K \to 2\pi$ and $K \to 3\pi$ amplitudes at $O(p^6)$ including the tree level, one- and two-loop diagrams. For the structure coefficients L_i of the effective chiral lagrangian at $O(p^4)$ we used the values fixed in [19] from the phenomenological analysis of low-energy meson processes. The structure coefficients of the effective chiral lagrangian at $O(p^6)$ have been fixed theoretically from the modulus of the logarithm of the quark determinant of the NJL-type model (see [17] for more details). The superpropagator regularization has been applied to fix UV divergences in meson loops, thereby the renormalization scale was $\mu_{SP} = 1$ GeV. The isotopic-symmetry-breaking corrections which arises from both the π^0-η and π^0-η' mixing

and the final-state $\pi^{\pm} - \pi^0$ mass difference were taken into account [2]). In [11] one can find more technical details of the calculation of $K \to 2\pi$ amplitudes.

4 Phenomenological analysis of $K \to 2\pi$ decays

In our phenomenological analysis the results on the $K \to 2\pi$ amplitudes, including the value for $\varepsilon^{'}/\varepsilon$, are compared with experiment to fix the phenomenological $\widetilde{B}_i$-factors for the mesonic matrix elements of nonpenguin and penguin four-quark operators. As experimental input we used the experimental values of the isotopic amplitudes $\mathcal{A}_{0,2}^{(exp)}$ fixed from the widths of $K \to 2\pi$ decays and the world average value $\mathrm{Re}\,\varepsilon^{'}/\varepsilon = (16.2 \pm 1.7) \times 10^{-4}$ which includes both old results of NA31 [21] and E731 [22] experiments and recent results from KTeV [23] and NA48 [24]. The output parameters of the performed $K \to 2\pi$ analysis are the factors $\widetilde{B}_1$, $\widetilde{B}_4$ and $\widetilde{B}_5$ for a fixed value of $\widetilde{B}_8$.

The dependence of the $\widetilde{B}_i$-factors on different theoretical uncertainties and experimental errors of various input parameters is investigated by applying the "Gaussian" method. Using Wilson coefficients derived in [7] in various regularization schemes (LO, NDR, HV) for different values of the QCD scale $\Lambda^{(4)}_{\overline{MS}}$,we calculated the probability density distributions for $\widetilde{B}_i$-factors obtained by using Gaussian distributions for all input parameters with their errors. As an example, the probability densities for the parameters $\widetilde{B}_1$, $\widetilde{B}_4$, $\widetilde{B}_5$ calculated with $\widetilde{B}_8 = 1$ and $\Lambda^{(4)}_{\overline{MS}} = 325$ MeV are shown in Figs. 1, 2 and 3. Upper and lower bounds for B_i-factors ($i = 1, 4, 5$) for different values of $\Lambda^{(4)}_{\overline{MS}}$ in LO, NDR and HV regularization schemes ($\widetilde{B}_8 = 1$) obtained by the Gaussian method are shown in table 1. The limits without parentheses correspond to the confidence level of 68% while the limits in parentheses – to the confidence level of 95%.

Fig. 3 shows the necessity for a rather large gluonic penguin contribution to describe the recently confirmed significant experimental $\varepsilon^{'}$ value (the factor $\widetilde{B}_5$ is found well above 1). It should be emphasized that the dependence of $\widetilde{B}_i$ ($i = 1, 4, 5$) on $\widetilde{B}_8$ is very small even within a wide range of its values $0 \le \widetilde{B}_8 \le 10$. Therefore even for $\widetilde{B}_8 = 0$ values of $\widetilde{B}_5 > 2$ are necessary to explain the large value of $\varepsilon^{'}/\varepsilon$.

Even for larger values of $\widetilde{B}_5$, the contributions of nonpenguin operators to the $\Delta I = 1/2$ amplitude are still dominating (see Fig. 4 and also upper and lower bounds for the relative contribution of penguin operators to the $\Delta I = 1/2$ amplitude in table 2). The large $\widetilde{B}_1$ and $\widetilde{B}_5$ values may be a hint that the long-distance contributions, especially to $\Delta I = 1/2$ amplitudes, are still not completely understood. An analogous conclusion has been drawn in [18], where also possible effects from physics beyond the Standard Model are discussed.

[2]) In [11] the π^0-η-$\eta^{'}$ mixing contribution to the isospin breaking has been computed only at tree level. The importance of the correction due to the final-state $\pi^{\pm} - \pi^0$ mass difference was discussed for the first time recently in [20]. This mass difference leads to a sizable correction to $\varepsilon^{'}/\varepsilon$, in spite of its smallness.

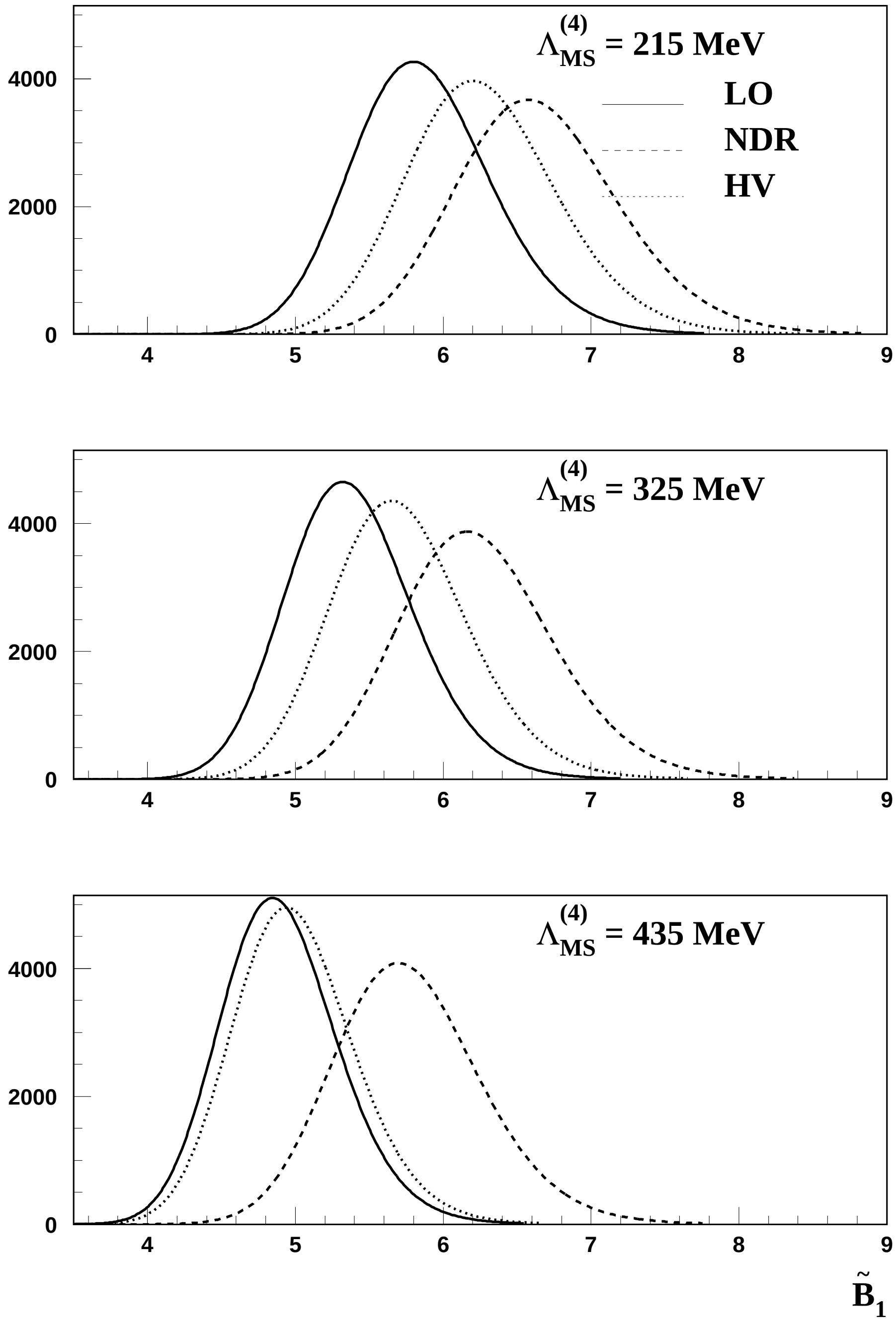

Fig. 1. Probability density distributions for factor $\widetilde{B}_1$ with $\widetilde{B}_8 = 1$

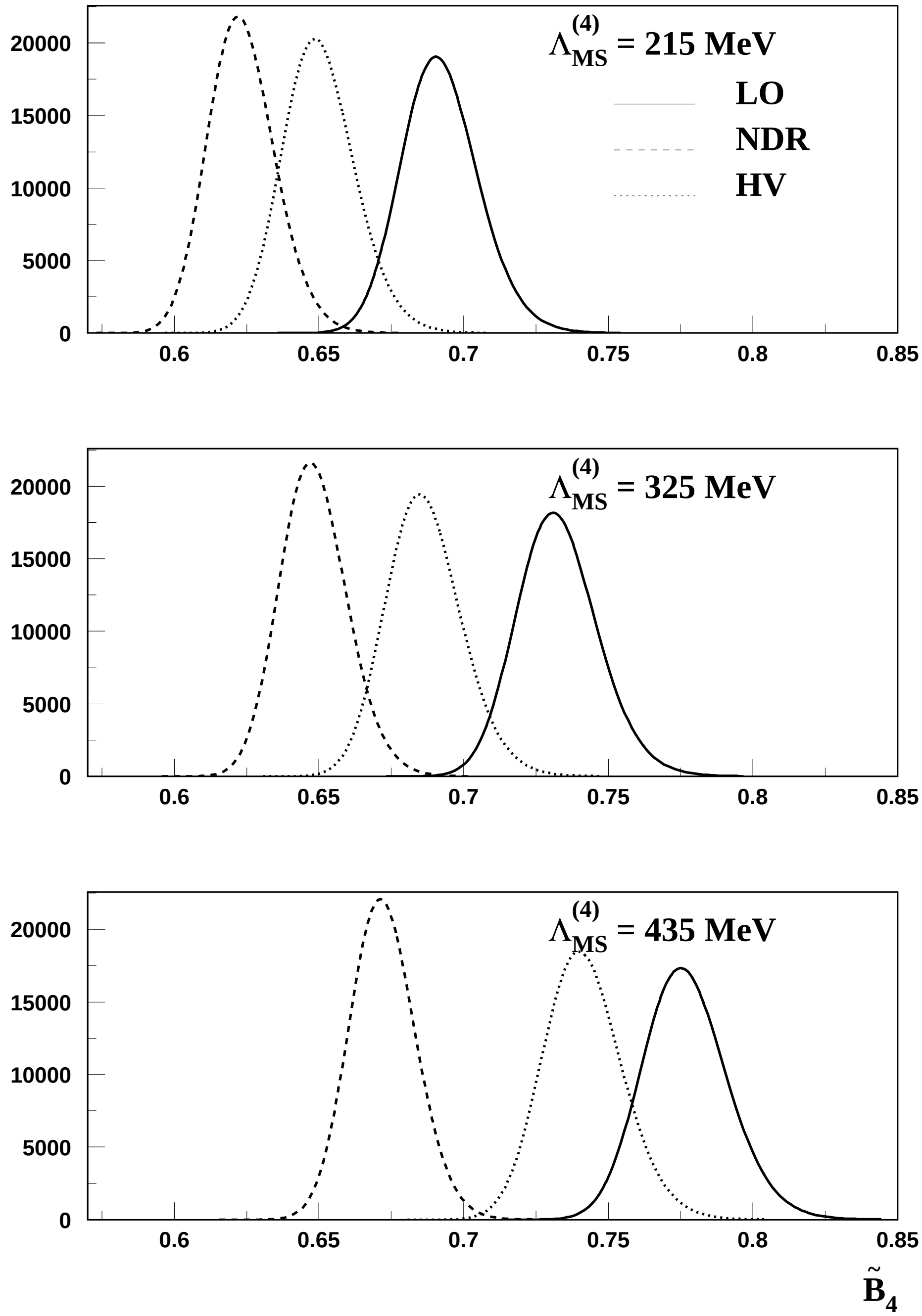

Fig. 2. Probability density distributions for factors $\widetilde{B}_4$ with $\widetilde{B}_8 = 1$

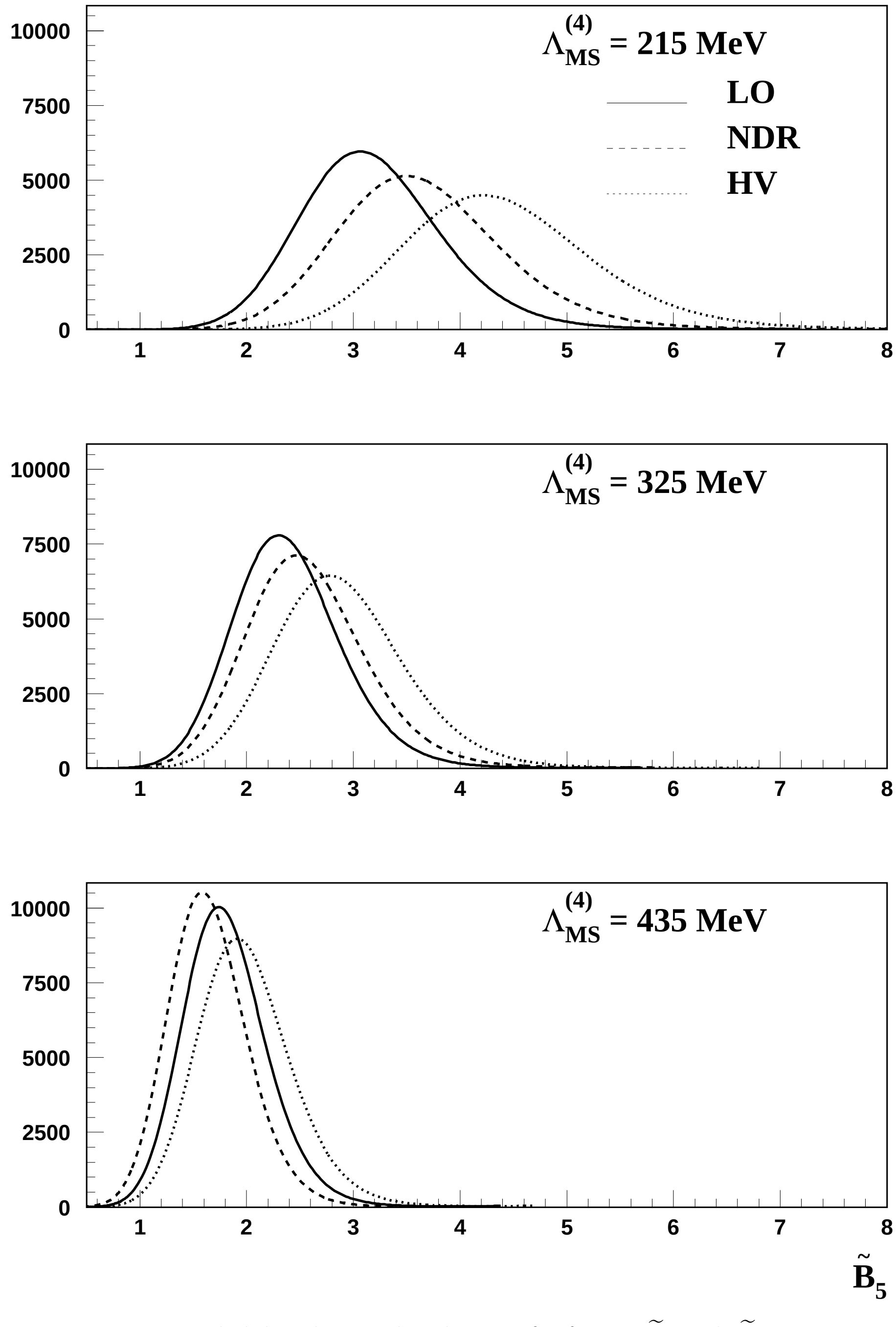

Fig. 3. Probability density distributions for factors $\widetilde{B}_5$ with $\widetilde{B}_8 = 1$

Table 1. Upper and lower bounds for $\widetilde{B}_i$ factors ($i = 1, 4, 5$). The limits without parentheses correspond to the confidence level of 68% while the limits in parentheses – to the confidence level of 95%.

B_i	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	LO min	LO max	NDR min	NDR max	HV min	HV max
$\widetilde{B}_1$	215	5.4	6.4	6.1	7.2	5.8	6.8
		(5.0	6.9)	(5.7	7.9)	(5.3	7.4)
	325	4.9	5.8	5.7	6.8	5.2	6.2
		(4.6	6.3)	(5.3	7.4)	(4.7	6.7)
	435	4.5	5.3	5.3	6.3	4.6	5.4
		(4.2	5.8)	(4.9	6.8)	(4.3	5.9)
$\widetilde{B}_4$	215	0.68	0.71	0.61	0.63	0.64	0.66
		(0.67	0.72)	(0.60	0.65)	(0.62	0.68)
	325	0.72	0.75	0.64	0.66	0.67	0.70
		(0.70	0.76)	(0.63	0.67)	(0.66	0.71)
	435	0.76	0.79	0.66	0.68	0.73	0.76
		(0.75	0.81)	(0.65	0.70)	(0.72	0.77)
$\widetilde{B}_5$	215	2.5	3.9	2.8	4.5	3.5	5.3
		(2.0	4.9)	(2.3	5.6)	(2.9	6.7)
	325	1.8	2.9	2.0	3.2	2.3	3.6
		(1.5	3.7)	(1.6	4.0)	(1.8	4.5)
	435	1.4	2.2	1.3	2.0	1.5	2.5
		(1.1	2.9)	(1.0	2.7)	(1.2	3.1)

5 Direct CP violation in $K \to 3\pi$ decays

Finally, the predictions for the CP asymmetry of linear slope parameters in the $K^\pm \to 3\pi$ Dalitz plot are discussed. These predictions are based on a new calculation of $K \to 3\pi$ amplitudes at $O(p^6)$ within the same effective lagrangian approach. The obtained $K \to 3\pi$ amplitudes include the same theoretical uncertainties as in case of the $K \to 2\pi$ analysis. The values of $\widetilde{B}_1$, $\widetilde{B}_4$, $\widetilde{B}_5$ fixed from the $K \to 2\pi$ analysis are used as phenomenological input to the $K \to 3\pi$ estimates which have been performed in a self-consistent way by the Gaussian method.

The linear slope parameter g of the Dalitz plot for $K \to 3\pi$ decays is defined through the expansion of the decay probability in the kinematic variables

$$|T(K \to 3\pi)|^2 \;\propto\; 1 + gY + \dots$$

where Y is a Dalitz variable:

$$Y = \frac{s_3 - s_0}{m_\pi^2}\,, \quad s_3 = (p_K - p_{\pi_3})^2\,, \quad s_0 = \frac{m_K^2}{3} + m_\pi^2\,,$$

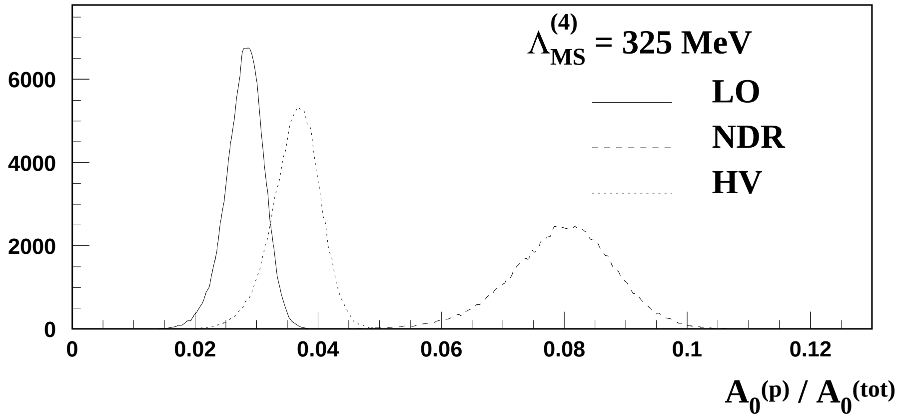

Fig. 4. Probability density distributions for the relative contribution of penguin operators to the $\Delta I = 1/2$ amplitude

Table 2. Upper and lower bounds for the relative contribution of penguin operators to the $\Delta I = 1/2$ amplitude. The limits without parentheses correspond to the confidence level of 68% while the limits in parentheses – to the confidence level of 95%.

	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	LO		NDR		HV	
		min	max	min	max	min	max
$A_0^{(p)}/A_0^{(tot)}$	215	0.023	0.029	0.062	0.078	0.031	0.039
		(0.019	0.031)	(0.053	0.084)	(0.027	0.042)
	325	0.025	0.031	0.071	0.088	0.032	0.040
		(0.021	0.034)	(0.061	0.096)	(0.027	0.044)
	435	0.026	0.033	0.079	0.098	0.043	0.054
		(0.022	0.036)	(0.067	0.107)	(0.037	0.059)

and the index π_3 belongs to the odd pion in the decays $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ and $K^\pm \to \pi^0\pi^0\pi^\pm$. Direct CP violation leads to a charge asymmetry of the linear slope parameter,

$$\Delta g(K^\pm \to 3\pi) = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)}\,.$$

Fig. 5 shows the probability density distributions for $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ and $K^\pm \to \pi^0\pi^0\pi^\pm$ decays calculated with $\widetilde{B}_8 = 1$ and $\Lambda^{(4)}_{\overline{MS}} = 325$ MeV. Upper and low bounds for Δg_{++-} and Δg_{00+} for different values of $\Lambda^{(4)}_{\overline{MS}}$ in LO, NDR and HV regularization schemes ($\widetilde{B}_8 = 1$) obtained by the Gaussian method are shown in table 3. Summarizing these results, we have obtained the following upper and lower

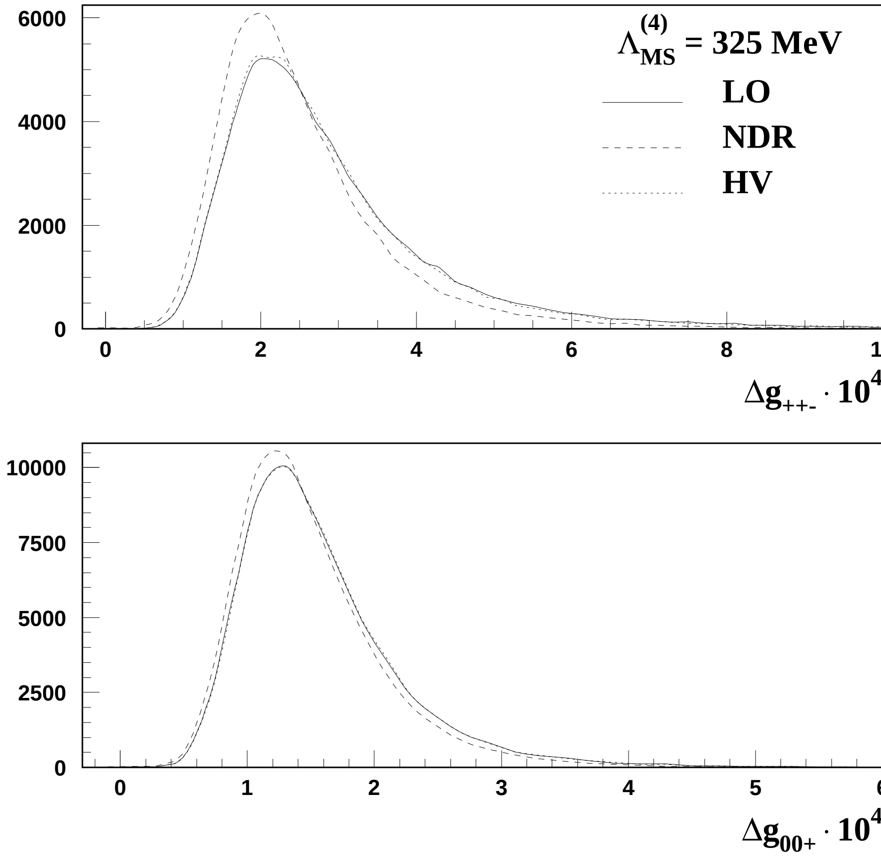

Fig. 5. Probability density distributions for the CP asymmetry of linear slope parameters of $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$ and $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ decays

bounds for the charge symmetries of the linear slope parameter:

$$1.6 \, < \, \Delta g_{++-} \cdot 10^4 \, < \, 4.2 \, , \quad 0.9 \, < \, \Delta g_{00+} \cdot 10^4 \, < \, 2.2 \;\; \text{with CL=68\%;}$$

$$1.1 \, < \, \Delta g_{++-} \cdot 10^4 \, < \, 7.6 \, , \quad 0.6 \, < \, \Delta g_{00+} \cdot 10^4 \, < \, 3.3 \;\; \text{with CL=95\%.}$$

When comparing these results with those found above for the phenomenological $\widetilde{B}$-factors, one finds a much reduced dependence on the regularization scheme and on the scale $\Lambda^{(4)}_{\overline{MS}}$. Once more this type of asymmetry ratios turns out to be more stable not only against systematic experimental errors (efficiencies, among others), but also with respect to theoretical uncertainties of its parts.

Table 3. Upper and low bounds for Δg_{++-} and Δg_{00+} (in units 10^{-4}). The limits without parentheses correspond to the confidence level of 68% while the limits in parentheses – to the confidence level of 95%.

Δg	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	LO min	LO max	NDR min	NDR max	HV min	HV max
Δg_{++-}	215	1.6	4.1	1.5	3.6	1.6	4.1
		(1.1	7.6)	(1.0	6.0)	(1.1	7.4)
	325	1.6	4.2	1.5	3.6	1.6	4.1
		(1.1	7.6)	(1.0	5.9)	(1.1	7.3)
	435	1.6	4.2	1.5	3.5	1.6	4.0
		(1.1	7.6)	(1.0	5.7)	(1.1	7.0)
Δg_{00+}	215	0.9	2.2	0.9	2.1	0.9	2.2
		(0.6	3.3)	(0.6	3.1)	(0.6	3.3)
	325	0.9	2.2	0.9	2.1	0.9	2.2
		(0.6	3.4)	(0.6	3.1)	(0.6	3.3)
	435	0.9	2.2	0.9	2.1	0.9	2.2
		(0.6	3.4)	(0.6	3.0)	(0.6	3.3)

6 Conclusion

With some experimental updates and theoretical refinements our new estimates confirm the dynamical enhancement mechanism for the charge asymmetry Δg by higher order contributions in the effective chiral lagrangian approach which was first observed in [9]. The predicted slope parameter asymmetry, although small, may be in reach of current high statistics experiments [25]. Already with lower statistics, new measurements of quadratic slope parameters of $K \to 3\pi$ decays, including the neutral channels, would lead to an improved theoretical understanding of the nonperturbative part of nonleptonic kaon decay dynamics.

For $\varepsilon^{'}/\varepsilon$, the results of analysis are very sensitive to final-state interactions, isotopic-symmetry-breaking effects and other refinements of the model calculations. Despite of considerable theoretical efforts dedicated to the calculation of Wilson coefficients and bag factors, the remaining uncertainties are still very large (compare also [26], where a similar conclusion is reached, when taking into account the full range of all input parameters and all theoretical uncertainties). The fact that phenomenological values of $\widetilde{B}_i$-factors ($i = 1, 4, 5$) considerably differ from unity shows that the long-distance contributions are still not completely understood. Besides the (rather remote) possibility, that the SM has to be revised in the kaon sector, there are further, more exotic developments in long-distance QCD going on [27], which may have manifestations also here. Another very active direction is the investigation of the problem of matching between short- and long-range renormalzation schemes by means of (effective) color singlet boson models. For a recent study,

especially of CP violation in $K \to 3\pi$ decays, see [28] and papers cited there.

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